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TRANSIENT RADIATIVE–CONDUCTIVE HEATING OF PLEXIGLAS

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Results of the numerical solution of the boundary-value problem of radiative–conductive heat transfer in a layer of Plexiglas are presented. The temperature fields in aircraft-cabin glazing are calculated.

Much attention is currently paid to the problem of joint heat transfer by heat conduction and radiation in semitransparent materials. This is caused by the necessity of solving some technological problems, in particular, studying the temperature fields in the case of heating and cooling of glasses. The results of a numerical study of radiative–conductive heat transfer in a selectively absorbing and radiating medium are presented in this paper.

We consider the formulation and the method of solving the problem of transient radiative–conductive heat transfer (RCHT) in a plane layer of SO-120 Plexiglas, which is used for glazing of aircraft cabins. The thermophysical properties of Plexiglas are assumed to be constant. The spectral coefficient of volume absorption at a temperature $T = 300$ K was calculated from the transmission spectrum of Plexiglas measured experimentally [1].

Heat transfer by conduction and radiation is described by the equation of energy with the initial and boundary conditions

$$\rho c_p \frac{\partial T}{\partial t} = \Lambda \frac{\partial^2 T}{\partial x^2} - \frac{\partial E}{\partial x}, \qquad 0 < x < L, \qquad t > 0, \tag{1}$$

$$\Lambda \frac{\partial T}{\partial x} = \alpha_1 (T - T_1) - \int\limits_{\Omega_1} \varepsilon_{1\nu} [Q_{1\nu}(T_1^*) - E_{1\nu}(T)]\, d\nu, \qquad x = 0 \tag{2}$$

$$\Lambda \frac{\partial T}{\partial x} = \alpha_2 (T_2 - T) + \int\limits_{\Omega_2} \varepsilon_{2\nu} [Q_{2\nu}(T_2^*) - E_{2\nu}(T)]\, d\nu, \qquad x = L, \tag{3}$$

$$T(x, 0) = T_0(x), \tag{4}$$

and also by the equations of energy transfer by radiation with the boundary conditions

$$\mu \frac{dI_\nu^+}{dx} + æ_\nu I_\nu^+ = æ_\nu I_{p\nu}(T), \quad \mu \frac{dI_\nu^-}{dx} - æ_\nu I_\nu^- = -æ_\nu I_{p\nu}(T), \quad 0 < x < L, \tag{5}$$

$$\begin{gathered} I_\nu^+(0, \mu) = (1 - R_{1\nu}) I_{p\nu}(T) + R_{1\nu} I_\nu^-(0, \mu), \\ I_\nu^-(1, \mu) = (1 - R_{2\nu}) I_{p\nu}(T) + R_{2\nu} I_\nu^+(L, \mu), \qquad 0 \leqslant \mu \leqslant 1. \end{gathered} \tag{6}$$

Here $I_{p\nu}(T) = 2\pi n^2 h\nu^3/(c_0^2[\exp(h\nu/(kT)) - 1])$, $E_{i\nu} = I_{p\nu}(T_i)$, $\mu = |\cos\varphi|$ (φ is the angle between the beam and the positive direction of the x axis), $æ_\nu$ is the volume coefficient of absorption of the material at a frequency ν, n is the refractive index, c_p is the specific heat capacity, ρ is the density of the medium, Λ is the thermal conductivity, L is the layer thickness, T_i and T_i^* are the temperatures of the ambient medium and external radiators, respectively, I_ν^+ and I_ν^+ are the spectral intensities of radiation in the positive and negative directions of the x axis, respectively, $I_{p\nu}$ is the Planck function, $Q_{i\nu}$, $E_{i\nu}$, $\varepsilon_{i\nu}$, $R_{i\nu}$, and Ω_i are the densities of the incident fluxes and intrinsic radiation,

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emissivities, reflection factors, and spectral regions of opaqueness of the boundary surfaces, respectively, and α_i are the coefficients of convective heat transfer at the boundaries ($i = 1, 2$).

Using the Green function

$$G(\xi, z) = \begin{cases} (\cosh z + \beta_1 \sinh z)[\cosh(1-\xi) + \beta_2 \sinh(1-\xi)]/\Delta, & z \leqslant \xi, \\ (\cosh \xi + \beta_1 \sinh \xi)[\cosh(1-z) + \beta_2 \sinh(1-z)]/\Delta, & z \geqslant \xi, \end{cases}$$

which is the solution of the uniform boundary-value problem [2]

$$\frac{\partial^2 G}{\partial z^2} - G = 0, \qquad \frac{\partial G}{\partial z} - \beta_1 G = 0 \quad (z = 0) \qquad \frac{\partial G}{\partial z} + \beta_2 G = 0 \quad (z = 1),$$

we reduce the initial-boundary problem (1)–(4) to a nonlinear integral equation with respect to the sought dimensionless temperature, which has the form

$$\theta(\xi, t) = q_1 G(\xi, 0) - q_2 G(\xi, 1) + \int_0^1 F(\theta) G(\xi, z)\, dz, \tag{7}$$

where

$$q_i = (-1)^i \Big\{ \beta_i \theta_i + \omega \int_{\Omega_i} \varepsilon_{i\nu} [Q_{i\nu}(\theta_i^*) - E_{i\nu}(\theta)]\, d\nu \Big\}, \quad F(\theta) = \omega \frac{\partial E}{\partial \xi} + R \frac{\partial \theta}{\partial t} - \theta(\xi, t),$$

$$\theta(\xi, t) = T(\xi, t)/T_*, \quad \theta_i^* = T_i^*/T_*, \quad G(\xi, 0) = [\cosh(1-\xi) + \beta_2 \sinh(1-\xi)]/\Delta,$$

$$G(\xi, 1) = [\cosh \xi + \beta_1 \sinh \xi]/\Delta, \quad \xi = x/L, \quad \theta_i = T_i/T_*, \quad \beta_i = \alpha_i L/\Lambda,$$

$$\omega = L/(\Lambda T_*), \quad R = L^2 \rho c_p/\Lambda, \quad \Delta = -(\beta_1 + \beta_2)\cosh 1 + (1 + \beta_1 \beta_2)\sinh 1 \quad (i = 1,\ 2),$$

and T_* is the characteristic temperature.

The radiation intensities, which are found from the solution of the boundary-value problem (5), (6) for the radiation-transfer equation, have the following form:

$$\begin{aligned} I_\nu^+(\xi, \mu) &= \Big[I_\nu^+(0, \mu) + \frac{\tau_\nu}{\mu} \int_0^\xi I_{p\nu}(y) \exp\Big(\frac{\tau_\nu}{\mu}\, y\Big) dy \Big] \exp\Big(-\frac{\tau_\nu}{\mu}\, \xi\Big), \\ I_\nu^-(\xi, \mu) &= \Big[I_\nu^-(1, \mu) + \frac{\tau_\nu}{\mu} \int_\xi^1 I_{p\nu}(y) \exp\Big(\frac{\tau_\nu}{\mu}\,(1-y)\Big) dy \Big] \exp\Big(-\frac{\tau_\nu}{\mu}\,(1-\xi)\Big). \end{aligned} \tag{8}$$

Using relations (6) and (8), we determine the boundary values of the intensities $I_\nu^+(0, \mu)$ and $I_\nu^-(1, \mu)$ by solving a system of algebraic equations:

$$I_\nu^+(0, \mu) = \Big\{ \varepsilon_1 I_{p\nu}(0) + R_{1\nu} \varepsilon_2 I_{p\nu}(1) \exp\Big(-\frac{\tau_\nu}{\mu}\Big) + \frac{\tau_\nu}{\mu} \int_0^1 I_{p\nu}(z) \Big[R_{1\nu} \exp\Big(-\frac{\tau_\nu}{\mu}\, z\Big) + R_{1\nu} R_{2\nu} \exp\Big(-\frac{\tau_\nu}{\mu}\,(2-z)\Big) \Big] dz \Big\} \Big/ D,$$

$$I_\nu^-(1, \mu) = \Big\{ \varepsilon_2 I_{p\nu}(1) + R_{2\nu} \varepsilon_1 I_{p\nu}(0) \exp\Big(-\frac{\tau_\nu}{\mu}\Big) + \frac{\tau_\nu}{\mu} \int_0^1 I_{p\nu}(z) \Big[R_{2\nu} \exp\Big(-\frac{\tau_\nu}{\mu}\,(1-z)\Big) + R_{1\nu} R_{2\nu} \exp\Big(-\frac{\tau_\nu}{\mu}\,(1+z)\Big) \Big] dz \Big\} \Big/ D.$$

Here $\varepsilon_1 = 1 - R_{1\nu}$, $\varepsilon_2 = 1 - R_{2\nu}$, and $D = 1 - R_{1\nu} R_{2\nu} \exp(2\tau_\nu/\mu)$.

It is shown by Ozisik [3] that

$$\frac{\partial E}{\partial \xi} = \int_0^\infty \tau_\nu [4 I_{p\nu}(\xi) - G_\nu(\xi)]\, d\nu,$$

where $\tau_\nu = æ_\nu L$ and

$$G_\nu(\xi) = 2 \int_0^1 \Big[I_\nu^+(0,\mu) \exp\Big(-\frac{\tau_\nu}{\mu}\,\xi \Big) + I_\nu^-(1,\mu) \exp\Big(-\frac{\tau_\nu}{\mu}\,(1-\xi) \Big) + \frac{\tau_\nu}{\mu} \int_0^1 I_{p\nu}(y) \exp\Big(-\frac{\tau_\nu}{\mu}\,|\xi - y| \Big) dy \Big] d\mu.$$

Thus, the RCHT problem (1)–(6) in a plane layer of a selectively absorbing and radiating medium reduces to solving [4] the nonlinear integral equation (7) relative to the sought dimensionless temperature $\theta(\xi, t)$ by an iterative method.

The integrals in (7) and (8) were calculated using the Gaussian quadrature formulas with 20 nodes, and the derivative $\partial\theta/\partial t$ was approximated by a finite-difference relation. The temperature profile was calculated for each time.

The numerical calculations, which took into account the selective character of radiation, were performed for SO-120 Plexiglas with the following thermophysical and optical parameters: $T_0 = 293$ K, $\Lambda = 0.183$ W/(m $\cdot$ K), $c_p = 1.73 \cdot 10^3$ J/(kg $\cdot$ K), $\rho = 1.1 \cdot 10^3$ kg/m^3, $T_* = 410$ K, and $n = 1.5$. We used the parameters of a typical flight of a supersonic cargo aircraft whose cruising velocity corresponds to the Mach number M = 2.2 [5].

Using the formulas derived in [6], we calculated the heat-transfer coefficient α_e and the recovery temperature T_e in the boundary layer for a point located at a distance of 1 m from the aircraft nose tip. The time dependence of velocity and flight altitude were used [5], and all the necessary parameters were assumed to be the same as in the "standard atmosphere." The dependences $\alpha_e(t)$ and $T_e(t)$ used in calculations of the convective heat flux on the outer surface of aircraft-cabin glazing are plotted in Fig. 1.

The effect of boundary-layer radiation on the outer surface of aircraft glazing was ignored, and the temperature and heat-transfer coefficient inside the cabin were $T_{\rm in} = 293$ K and $\alpha_{\rm in} = 5$ W/(m$^2 \cdot$ K), respectively. Re-radiation inside the layer of Plexiglas and own radiation and convective heat fluxes on the glass surfaces were taken into account. In this case, the boundary conditions (2) and (3) acquire the following form:

$$\Lambda \frac{\partial T}{\partial x} = \alpha_e(t)(T - T_e(t)) + \int_{\Omega_1} \varepsilon_{1\nu} E_{1\nu}(T)\, d\nu \qquad (x = 0),$$

$$\Lambda \frac{\partial T}{\partial x} = \alpha_{\rm in}(T_{\rm in} - T) - \int_{\Omega_2} \varepsilon_{2\nu} E_{2\nu}(T)\, d\nu \qquad (x = L).$$

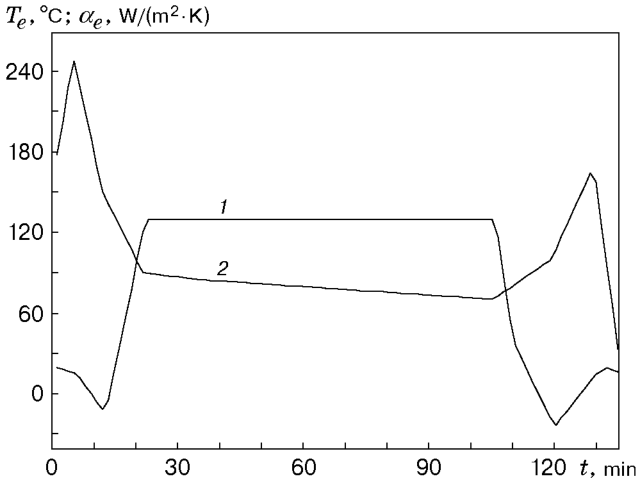
Figures 2–8 show the calculation results of the process of heating of Plexiglas with the following optical characteristics: $\varepsilon_{1\nu} = 1$, $\varepsilon_{2\nu} = 0$, $R_{1\nu} = 0$, and $R_{2\nu} = 1$.

Figure 2 shows the time dependence of the temperature on the outer and inner surfaces of aircraft-cabin glazing for two thicknesses, with internal re-radiation in the Plexiglas layer taken into account ($\varepsilon_{1\nu} = 1$, curves 1 and 3) and ignored ($\varepsilon_{1\nu} = 0$, curves 2 and 4). It follows from Fig. 2 that taking into account internal re-radiation in the layer leads to a decrease in temperature both on the inner and outer surfaces of cabin glazing, which may be attributed to the effect of intrinsic radiation of the layer surface. It should be noted that the temperatures at the outer surface of glasses differ insignificantly for both thicknesses. A significant difference in temperature is observed on the inner surface for $h = 0.01$ m and $h = 0.03$ m.

Figure 3 shows the temperature difference $\Delta T = T_1 - T_2$ as a function of time (T_1 and T_2 are the temperatures with radiation transfer in the Plexiglas layer ignored and taken into account, respectively).

Figures 4–6 show the calculation results for the temperature distributions with ignored re-radiation in the Plexiglas layer for different times (from aircraft take-off to landing). It follows from Fig. 4 that the temperature in the Plexiglas layer decreases over the time interval of 5–15 min. Then, the temperature increases over the interval of 17.5–25 min (beginning of the cruising regime) (Fig. 5). The temperature distribution in the regime of aircraft landing is plotted in Fig. 6. As in Fig. 4, a decrease in temperature with time is observed. These features of the temperature regime in the layer are explained by the behavior of the dependences $T_e(t)$ and $\alpha_e(t)$ (see Fig. 1).

Figures 7 and 8 show the temperature profiles taking into account intrinsic radiation in the Plexiglas layer in the take-off, cruising, and landing regimes. The same features in the temperature-field changes are observed, as in the case with ignored intrinsic radiation (see Figs. 5 and 6).



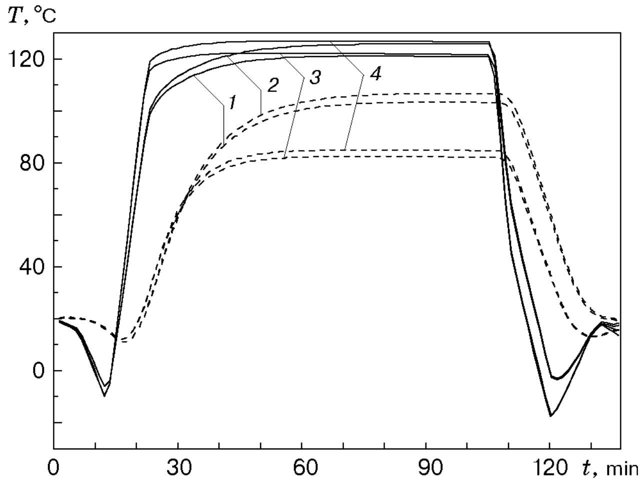
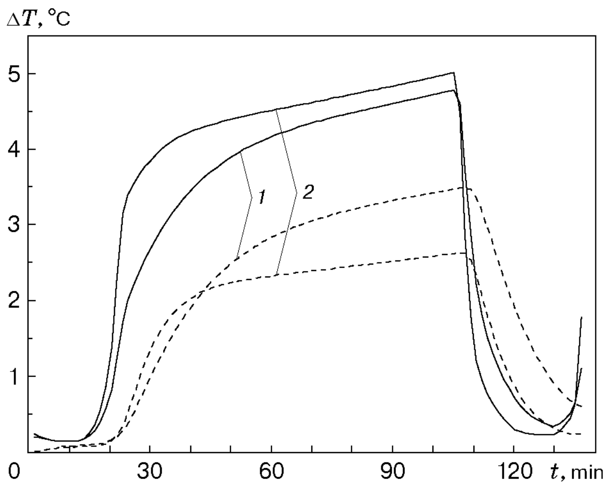


Fig. 1 Fig. 2

Fig. 1. Time dependences of the recovery temperature (1) and heat-transfer coefficient (2) on the outer surface of cabin glazing.

Fig. 2. Time dependences of the temperature on the outer (solid curves) and inner (dashed curves) surfaces of cabin glazing with internal re-radiation in the Plexiglas layer taken into account (curves 1 and 3) and ignored (curves 2 and 4) for $h = 0.01$ (curves 1 and 2) and 0.03 m (curves 3 and 4).



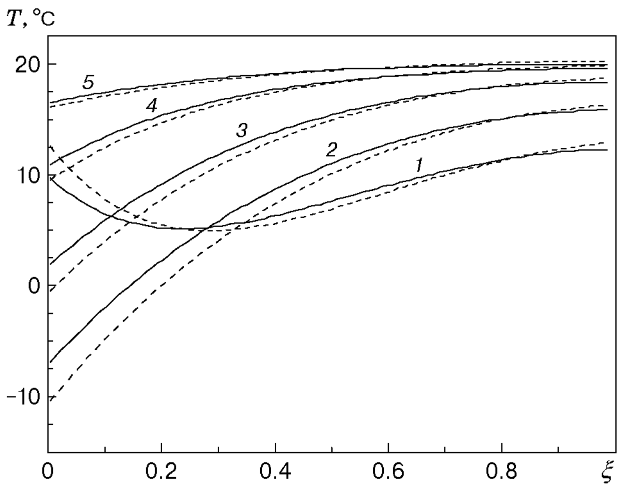


Fig. 3 Fig. 4

Fig. 3. Time dependence of the temperature difference (with internal re-radiation in the Plexiglas layer taken into account and ignored) on the outer (solid curves) and inner (dashed curves) surfaces of cabin glazing for $h = 0.01$ (1) and 0.03 m (2).

Fig. 4. Temperature distribution with ignored internal re-radiation in the Plexiglas layer in the take-off regime for $h = 0.01$ (solid curves) and 0.03 m (dashed curves) and $t = 5$ (1), 7.5 (2), 10 (3), 12.5 (4), and 15 min (5).

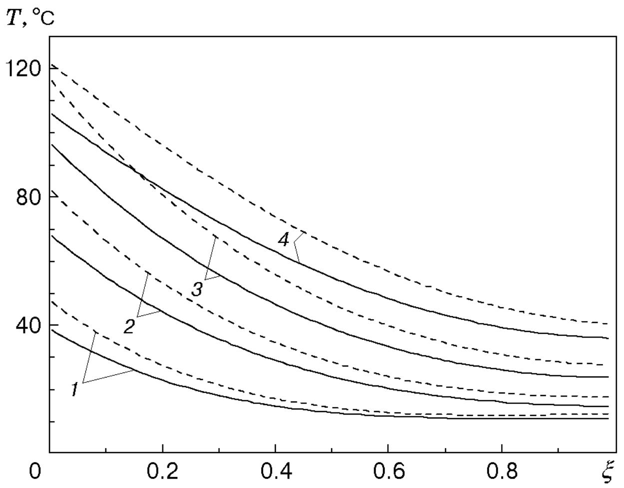


Fig. 5

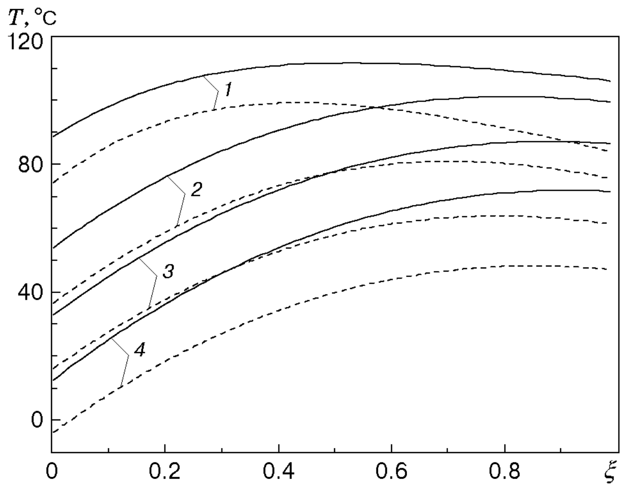


Fig. 6

Fig. 5. Temperature distribution with ignored internal re-radiation in the Plexiglas layer in the cruising flight regime of the aircraft for $h = 0.01$ (solid curves) and 0.03 m (dashed curves) and $t = 17.5$ (1), 20 (2), 22.5 (3), and 25 min (4).

Fig. 6. Temperature distribution with ignored internal re-radiation in the Plexiglas layer in the aircraft-landing regime for $h = 0.01$ (solid curves) and 0.03 m (dashed curves) and $t = 109$ (1), 112 (2), 115 (3), and 118 min (4).

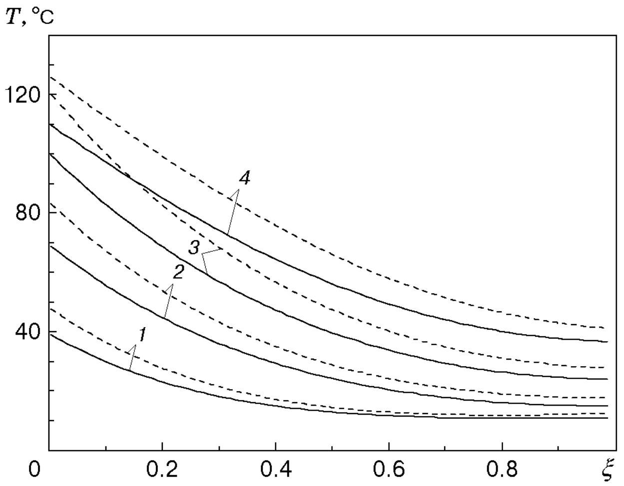


Fig. 7

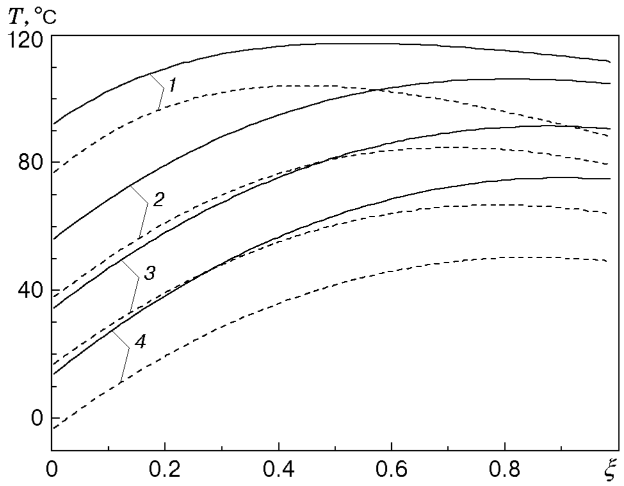


Fig. 8

Fig. 7. Temperature distribution taking into account internal re-radiation in the Plexiglas layer in the cruising flight regime of the aircraft (notation the same as in Fig. 5).

Fig. 8. Temperature distribution taking into account internal re-radiation in the Plexiglas layer in the aircraft-landing regime (notation the same as in Fig. 6).

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